# Gaussian Process Conjoint Analysis for Adaptive Marginal Effect Estimation

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## Abstract

Choice-based conjoint analysis is an essential tool for learning the marginal effects of multidimensional explanatory features on preferences. However, existing marginal effect models rely on either non-parametric estimators that generalize poorly to individualized effects, or linear latent utility that completely ignores possible high-order interactions. We introduce Gaussian process conjoint analysis (GPCA) for learning marginal effects from observed choices as the first-order derivatives of the unknown systems. We also propose Gaussian mixture approximation for the predictive distributions of marginal effects that facilitates downstream tasks such as adaptive experimentation. Through both synthetic and real data, we show GPCA achieves more precise estimation of marginal effects and higher efficiency of effect estimation using adaptive experimentation.

## 1 Introduction

Understanding the relationship between targeted outcomes and features in survey experiments is fundamental in many disciplines such as social science [1–3], human-computer interaction [4, 5] and marketing research [6–8]. These associations are often captured by marginal effect, defined as the change in predicted outcomes resulting from changes in features. Depending on the type of attributes, marginal effects could either be computed as the discrete change in outcomes for categorical attributes or infinitesimal margins for continuous attributes. In survey experiments, marginal effects are often learned using the choice-based conjoint experiments which present a series of profile pairs at varying attribute values so as to compare the difference in averaged outcomes [6]. For example, researchers alternate background characteristics to study bias towards immigrants, and system designers change interface setups to improve click-through rates of their new web interface.

However, learning marginal effects from conjoint analysis encounters several challenges. First, effects of a single attribute may be heterogeneous when interacting with other attributes. To learn possible heterogeneous effects caused by high-order interactions, existing methods usually rely on stacking multiple attributes in a difference-in-different style that makes estimation of interaction effects involving more attributes extremely complicated [1, 2]. Alternatively, marginal effects may also be captured by the first-order derivatives of attributes w.r.t to the preference outcomes through a latent utility function. However, previous work typically depend on linear models such as support vector machine to learn partial utilities that overlooks possible interactions in the feature space [8–11].

Second, the multi-dimensional nature in conjoint experiments may lead to small-sample biases in effect estimation, as common randomization design would inevitably split sample sizes on each level of attributes. Hence, adaptive experimentation may be needed for acquiring next pairs of profiles when querying of unknown preferences is expensive. By utilizing prior responses and maintaining a belief model of the system, adaptive experimentation could balance between exploiting attributes that are more crucial to the preference and exploring attributes that the model is uncertain about.

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In this work, we study the problem of marginal effect estimation in choice-based conjoint analysis and propose Gaussian process conjoint analysis (GPCA) that automatically learns high-order interactions by using the preference learning framework. We derive marginal effects using first-order derivatives of Gaussian process learned from observed preferences, and approximate the distributions of marginal effects via Gaussian mixture models. By building a predictive model of the latent system, GPCA could also facilitate adaptive experimentation such as Bayesian active learning by disagreement to accelerate effect estimation. As shown in the simulated experiments, GPCA is able to achieve more precise estimation of marginal effects than other non-parametric and parametric methods. Finally, we apply GPCA to two real-world online experiments: learning citizens' preferences across presidential candidates and examining attitudes toward immigrants.

## 2 Related work

**Conjoint analysis.** Originally introduced as a marketing tool [6, 7], conjoint analysis has been used for learning multi-dimensional treatment effects using non-parametric estimators in quantitative research [1, 2] or eliciting user preferences in recommendation system via parametric utility functions [10, 12]. Hainmueller et al. [1] proposed a difference-in-difference interaction effect estimator for eliciting preferences from multi-dimensional choices in survey experiments, where the inner and outer differences come from the target and interacted attributes. Subsequently, Egami and Imai [2] proposed a new effect estimator in factorial experiments that does not depend on the choice of baseline conditions and generalizes better for higher-order interaction effects. However, these work focus on discrete attributes and have to categorize continuous attributes into distinct subgroups that are subject to categorization. Alternatively, Chapelle and Harchaoui [10] introduced a generalized logistic approach by learning a parametric latent utility and explaining observed preferences via a softmax function. Similar utility-based methods include support vector machines [9, 8, 11], Gaussian processes [13, 12, 14–16] and decision trees [17]. However, these preference learning methods emphasize learning the most preferred recommendations through latent utilities of low interpretability, rather than estimation of marginal effects that explains the relation between attributes and outcomes.

**Marginal effects.** Marginal effects was studied in economics for measuring the responsiveness of economic variables by the concept of elasticity [18], for instance, how the percentage of demand quantity falls due to percentage of change in price. Hence, marginal effects are often used for understanding transformed features in regression models [19] or examining heterogeneous association between feature and outcomes [20]. Another stream of work focus on using marginal effects for machine learning model interpretability. Silva Filho et al. [21] provided a feature importance method for interpreting classification models based on marginal local effects. Merz et al. [22] proposed a marginal attribution method by conditioning on quantiles for analyzing global gradients in deep neural network. Scholbeck et al. [17] introduced forward marginal effects that unify and mixed-type features as a general model-agnostic interpretation method for general non-linear machine learning models. However, marginal effects in preference learning has not been investigated in these literature.

**Adaptive experiment.** Often framed as a sequential decision making or active learning problem [23], adaptive experimentation utilizes already collected responses for informing experiment setup or data acquisition in next iterations to maximize the usefulness of limited data. Adaptive experiment has been adopted by domain scientists to accelerate scientific discovery. For instance, Bayesian optimization via adaptive sample selection were successfully applied in material science for discovering new materials [24] and clinical trials for finding maximum tolerated dose [25, 26]. Meanwhile, active search was introduced for iterative design of virtual screening trials in chemoinformatics [27]. In machine learning, Chen et al. [28] studied the pairwise ranking problem in crowd-sourcing setup with online learning. Bıyık et al. [29] proposed an active preference-based learning based on information gain for reward functions in robotics. However, previous adaptive designs in quantitative research have been mainly focused on treatment selection in bandit settings [30–32], with limited attention to marginal effect estimation particularly within the GP preference learning framework.

## 3 Backgrounds

**Notations.** Formally, let $\mathbf{x} \subseteq \mathbb{R}^d$ denote all $d$-dimensional attributes of the full profile, and $\mathbf{x}_l$ represents the $l$th attribute and $\mathbf{x}_{-l}$ represents the remaining attributes other than the $l$th. Furthermore, for pairwise comparison, let $y_{ij} \in \{0,1\}$ denote whether the left-side profile $\mathbf{x}^{(i)}$ is preferred to the right-side $\mathbf{x}^{(j)}$, where $y_{ij} = 1$ if $\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)}$ and $y_{ij} = 0$ otherwise. Here we focus on choice-based conjoint analysis with pairwise comparison, as multiple choices could be easily transformed into multiple comparison of pairs. For instance, $\mathbf{x}^{(i)}$ is mostly preferred amongst $\{\mathbf{x}^{(i)}, \mathbf{x}^{(j)}, \mathbf{x}^{(k)}\}$ is equivalent to $\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)}$ and $\mathbf{x}^{(i)} \succ \mathbf{x}^{(k)}$. Our notation could also account for score-based conjoint experiments, where $\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)}$ could indicate $\mathbf{x}^{(i)}$ having higher score than $\mathbf{x}^{(j)}$. Furthermore, suppose all revealed preferences are collected into $\mathcal{D} = \{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}), y_{ij}\}$.

**Marginal effects of discrete attributes.** In conjoint analysis with factorial design, attributes usually take discrete values of different levels $\mathbf{x}_l = 1, \ldots, C_l$. For a target distribution of profiles $\mathcal{P}$, the marginal effects $\pi_l(a,b)$ of attribute $\mathbf{x}_l$ from level $a$ to $b$ ($1 \le a < b \le C_l$) are captured by the average marginal component effect (AMCE), defined as the difference in expected preferential outcomes averaged over all the possible values of the remaining attributes $\mathbf{x}_{-l}$ over $\mathcal{P}$:

$$\pi_l(a,b) = \mathbb{E}_{\mathbf{x}^{(i)}_{-l}, \mathbf{x}^{(j)} \sim \mathcal{P}}[y_{ij} \mid \mathbf{x}^{(i)}_l = b] - \mathbb{E}_{\mathbf{x}^{(i)}_{-l}, \mathbf{x}^{(j)} \sim \mathcal{P}}[y_{ij} \mid \mathbf{x}^{(i)}_l = a] \tag{1}$$

Intuitively, $\pi_l(a,b)$ represents the increase in the probability of one profile being preferred if the $l$th attribute were $b$ instead of $a$ for profile distribution $\mathcal{P}$. With the conditionally independent assumption, $\pi_l(a,b)$ can be estimated straight-forwardly using a difference-in-mean approach:

$$\hat{\pi}_l(a,b) = \frac{\sum_{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \in \mathcal{D}} y_{ij} \mathbb{I}[\mathbf{x}^{(i)}_l = b]}{\sum_{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \in \mathcal{D}} \mathbb{I}[\mathbf{x}^{(i)}_l = b]} - \frac{\sum_{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \in \mathcal{D}} y_{ij} \mathbb{I}[\mathbf{x}^{(i)}_l = a]}{\sum_{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \in \mathcal{D}} \mathbb{I}[\mathbf{x}^{(i)}_l = a]} \tag{2}$$

However, this difference-in-mean approach for estimating marginal effects suffers from two issues. First, generalization of this estimator for heterogeneous effect resulting from either background characteristics or high-level interactions could get more complicated as calculation of multiple differences is required. For instance, for obtaining interaction effects of between $\mathbf{x}^{(i)}_l$ and $\mathbf{x}^{(i)}_m$ from level $c$ to $d$ in $\mathbf{x}^{(i)}_m$, one needs to compute $[\hat{\pi}_l(a,b)\big|_{\mathbf{x}^{(i)}_m = c} - \hat{\pi}_l(a,b)\big|_{\mathbf{x}^{(i)}_m = c}] - [\hat{\pi}_l(a,b)\big|_{\mathbf{x}^{(i)}_m = d} - \hat{\pi}_l(a,b)\big|_{\mathbf{x}^{(i)}_m = d}]$ [1]. Second, in practice, continuous attributes are rarely repeated and thus often need to be discretized into multiple levels; otherwise, each level $\mathbf{x}^{(i)}_l = a$ would have very few observations. However, this discretization is subject to the chosen cutoff points and may lead to an oversimplification of the system, threatening the internal validity of marginal effect estimation.

# 4 Gaussian process conjoint analysis

We now introduce Gaussian process conjoint analysis (GPCA) for estimating marginal effects in conjoint analysis of mixed-type attributes. We then derive marginal effects in GPCA and propose the use of Gaussian mixture model for effectively approximating their distributions.

## 4.1 Preference learning with Gaussian process

Conjoint analysis can also be framed as a preference learning problem with a latent utility function $u(\mathbf{x})$ that takes mixed-type attributes. The preferential relation between $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ is then determined by comparing their utilities $u(\mathbf{x}^{(i)})$ and $u(\mathbf{x}^{(j)})$. Through a sigmoid probabilistic model $\sigma(\cdot)$, the probability of observed preference $p(\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)}) = \sigma\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})\big)$ could also allow possible labeling error. Gaussian process (GP) preference learning places a GP prior on latent utility $u(\mathbf{x}) \sim \mathcal{GP}(0, K)$ with RBF kernel $K(x,x') = \exp(-\|x - x'\|^2/2)$, and uses a cumulative standard normal function for observation model $p\big(\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)} \mid u(\mathbf{x}^{(i)}), u(\mathbf{x}^{(j)})\big) = \Phi\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})\big)$. Although the posterior of $u(\mathbf{x})$ is no longer analytical for GP classification, it could be approximated using standard methods such as Laplace approximation and expectation propagation [33, 12].

Furthermore, the inferred latent utility posterior could also be used for prediction. For any new pair of profiles $(\mathbf{x}^{(i)}_*, \mathbf{x}^{(j)}_*)$, suppose their corresponding utility vector has been approximated by a bivariate normal $\mathbf{u}_* = [u(\mathbf{x}^{(i)}_*), u(\mathbf{x}^{(j)}_*)]^T \sim \mathcal{N}(\boldsymbol{\mu}_*, \boldsymbol{\Sigma}_*)$. Let $\boldsymbol{\mu}_* = [\mu^{(i)}_*, \mu^{(j)}_*]^T$ and $\sigma^2_* =$

$1 + [1, -1]\mathbf{\Sigma}_*[1, -1]^T$, then the predictive probability has the following closed-form:

$$p(\mathbf{x}_*^{(i)} \succ \mathbf{x}_*^{(j)}) = \int \Phi\big(u(\mathbf{x}_*^{(i)}) - u(\mathbf{x}_*^{(j)})\big) p(\mathbf{u} \mid \mathcal{D}) d\mathbf{u} = \Phi(\frac{\mu_*^{(i)} - \mu_*^{(j)}}{\sigma_*}) \tag{3}$$

Sometimes the predictive probability in Eq. (3) are directly defined on pairs of profiles $(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ using preference kernel. As the difference of two Gaussians remains Gaussian, a GP on $u(\mathbf{x}^{(i)})$ will also induce a GP on $u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})$ but with a preference kernel $K_{\text{pref}}\big((\mathbf{x}_1^{(i)}, \mathbf{x}_1^{(j)}), (\mathbf{x}_2^{(i)}, \mathbf{x}_2^{(j)})\big) = K(\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(i)}) - K(\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(j)}) - K(\mathbf{x}_2^{(i)}, \mathbf{x}_1^{(j)}) + K(\mathbf{x}_1^{(j)}, \mathbf{x}_2^{(j)})$. We adopted this preference kernel in our implementation of GPCA.

### 4.2 Marginal effects in GPCA

We follow the definition of AMCE in Eq. (1) but adapted to our GPCA framework. We exploit the affine property of Gaussian processes to derive marginal effects of mixed-type attributes using first-order gradients, where discrete attributes can be converted to continuous attributes with additional dummy variables. Our discussion will focus on marginal effects of profile pairs $(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ on both sides. Specifically, the gradient $\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big)$ in probability of target profile $\mathbf{x}^{(i)}$ being preferred to opponent profile $\mathbf{x}^{(j)}$ can be derived as:

$$\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big) = \frac{\partial}{\partial(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})}[p(\mathbf{x}^{(i)} \succ \mathbf{x}^{(j)})] \qquad \text{definition of AMCE} \tag{4}$$

$$= \frac{\partial}{\partial(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})}\mathbb{E}_{u|\mathcal{D}}\Big[\Phi\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})\big)\Big] \qquad \text{averaged by } u \mid \mathcal{D} \tag{5}$$

$$= \mathbb{E}_{u|\mathcal{D}}\Big[\phi\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})\big)\big(\nabla u(\mathbf{x}^{(i)}), -\nabla u(\mathbf{x}^{(j)})\big)\Big] \qquad \text{chain rule} \tag{6}$$

Note that in the second step we swapped the order of expectation and differentiation. Intuitively, the marginal effects of $(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ on the outcome space can be computed as the expected gradient $\big(\nabla u(\mathbf{x}^{(i)}), -\nabla u(\mathbf{x}^{(j)})\big)$ in the latent utility space further weighted by the probability densities $\phi\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})\big)$ of a normal distribution at the latent utility distance $u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})$. For the sake of notation, further denote the one-sided marginal effect $\phi\big(u(\mathbf{x}) - u(\mathbf{x}^{(j)})\big)\nabla u(\mathbf{x})$ as $\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D})$ where $\mathcal{D}$ indicates the posterior of utility on $\mathcal{D}$. Since the normal pdf is symmetric, we could conveniently write the right-side gradient as $-\phi\big(u(\mathbf{x}^{(i)}) - u(\mathbf{x})\big)\nabla u(\mathbf{x}) = -\phi\big(u(\mathbf{x}) - u(\mathbf{x}^{(i)})\big)\nabla u(\mathbf{x})$ as $-g(\mathbf{x}; \mathbf{x}^{(i)}, \mathcal{D})$ and hence marginal effect as $\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big) = \big(\mathbf{g}(\mathbf{x}^{(i)}; \mathbf{x}^{(j)}, \mathcal{D}), -\mathbf{g}(\mathbf{x}^{(j)}; \mathbf{x}^{(i)}, \mathcal{D})\big)$. Lastly, $\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big)$ captures the concatenated multi-variate distribution of marginal effects for the entire profile vectors, and could be easily projected along any unit vector $\hat{\mathbf{e}}_l$ to obtain the component effects analogous to Eq. (1). Intuitively, component effects represent the attribute-specific effects on preferences, averaged over profile population:

$$\pi_l(\mathbf{x}_l^{(i)}) = \sum_{(\mathbf{x}_{-l}^{(i)}, \mathbf{x}^{(j)}) \sim \mathcal{P}} \langle \pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big), \hat{\mathbf{e}}_l \rangle \tag{7}$$

### 4.3 Gaussian mixture approximation of marginal effects

As $\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big)$ involves taking weighted averages of utility gradient vector $\nabla u(\mathbf{x})$ over utility posterior $u \mid \mathcal{D}$, we propose the use of Gaussian mixture model (GMM) to approximate its distribution. As the gradient of a GP is still a GP, we can first write the joint distribution of utility $u(\cdot) \mid \mathcal{D}$ and utility gradient $\nabla u \mid \mathcal{D}$ under utility posterior $\mathcal{GP}\big(\mu_{u|\mathcal{D}}(\mathbf{x}), K_{u|\mathcal{D}}(\mathbf{x}, \mathbf{x}')\big)$ on $\mathcal{D}$ as:

$$\begin{bmatrix} u \mid \mathcal{D} \\ \nabla u \mid \mathcal{D} \end{bmatrix} \sim \mathcal{GP}\Big( \begin{bmatrix} \mu_{u|\mathcal{D}} \\ \nabla \mu_{u|\mathcal{D}} \end{bmatrix}, \begin{bmatrix} K_{u|\mathcal{D}} & \nabla K_{u|\mathcal{D}}{}^T \\ \nabla K_{u|\mathcal{D}} & \nabla^2 K_{u|\mathcal{D}} \end{bmatrix} \Big) \tag{8}$$

where $\nabla \mu_{u|\mathcal{D}} = \partial \mu_{u|\mathcal{D}}(\mathbf{x})/\partial \mathbf{x}$ is the first-order derivative of the posterior mean, $\nabla K_{u|\mathcal{D}} = \partial K_{u|\mathcal{D}}(\mathbf{x}, \mathbf{x}')/\partial \mathbf{x}$ is the first-order derivative of the posterior covariance and $\nabla^2 K_{u|\mathcal{D}} = \partial^2 K_{u|\mathcal{D}}(\mathbf{x}, \mathbf{x}')/\partial \mathbf{x} \partial \mathbf{x}'$ is its second-order mixed derivatives.

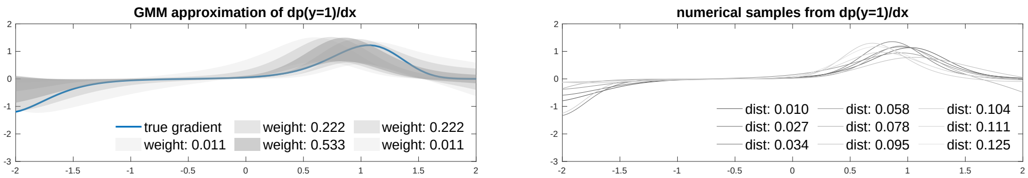



Figure 1: Visualization of the proposed GMM for approximating one-side marginal effect. Left figure shows our GMM approximation of the one-side marginal effect using 5 sampling points, and right figure shows 9 possible true effects obtained by numerical sampling. Darker colors indicate components with higher weights in the GMM and numerical samples closer to the one-side marginal effect posterior mode.

Although the joint distribution in Eq. (8) is Gaussian, the one-sided marginal effect $\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D})$ is not because it involves the product of a multivariate Gaussian $\nabla u(\mathbf{x}) \mid \mathcal{D}$ and a non-linear transformation $\phi(\cdot)$ of an univariate Gaussian $u(\mathbf{x}) - u(\mathbf{x}^{(j)}) \mid \mathcal{D}$. Therefore, we use a Gaussian mixture model (GMM) to approximate $\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D})$. Each component of the GMM is formed by scaling the multivariate Gaussian with the transformed values of quadrature points of the univariate Gaussian determined by Gauss-Hermite quadrature. Let $N$ be the number of points in the quadrature, $k_r$ be the roots of the physicists' version of the Hermite polynomial $H_N(k)$ and $\omega_r = \frac{2^{N-1}N!}{N^2[H_{N-1}(k_r)]^2}$ be the weights of each component [34]. We could then approximate $\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D})$ as:

$$\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D}) \approx \sum_{r=1}^{N} \omega_r \phi\big(\bar{f}_r(\mathbf{x})\big) \circ \mathcal{N}\Big(\nabla \mu_{u|\mathcal{D}}(\mathbf{x}), \nabla^2 K_{u|\mathcal{D}}(\mathbf{x}, \mathbf{x})\Big) \tag{9}$$

$$= \sum_{r=1}^{N} \omega_r \mathcal{N}\Big(\phi\big(\bar{f}_r(\mathbf{x})\big) \circ \nabla \mu_{u|\mathcal{D}}(\mathbf{x}), \phi\big(\bar{f}_r(\mathbf{x})\big)\phi\big(\bar{f}_r(\mathbf{x})\big)^T \circ \nabla^2 K_{u|\mathcal{D}}(\mathbf{x}, \mathbf{x})\Big) \tag{10}$$

where $\bar{f}_r(\mathbf{x}) = \sqrt{2}[\sigma^2_{u|\mathcal{D}}(\mathbf{x}) + \sigma^2_{u|\mathcal{D}}(\mathbf{x}^{(j)})]^{1/2} k_r + [\mu_{u|\mathcal{D}}(\mathbf{x}) - \mu_{u|\mathcal{D}}(\mathbf{x}^{(j)})]$ are locations of mixture components defined on the sample point $k_r$s, and $\circ$ denotes the Hadamard (element-wise) product. Figure 1 shows the visualization of the proposed GMM for approximating one-side marginal effect. The left-hand side shows our GMM approximation of the one-sided marginal effect using 5 sampling points, and the right-hand side shows 9 possible true effects obtained by numerical sampling. Darker colors indicate components with higher weights in the GMM and numerical samples closer to the one-side marginal effect posterior mode. We found in experiments with just $N = 10$ quadrature points, our GMM was able to effectively approximate the true distribution of $\mathbf{g}(\mathbf{x}; \mathbf{x}^{(j)}, \mathcal{D})$.

# 5 Adaptive experimentation in GPCA

We investigate the use of adaptive experimentation with GPCA to acquire the most informative pairs of profiles for estimating marginal effects. Informed by the posterior belief on the latent utility, adaptive experimentation may efficiently explore attributes whose marginal effects on preferences are less certain. To this end, we can determine the next pairs of profiles to compare by maximizing an *acquisition function* $(\mathbf{x}^{(i)}_*, \mathbf{x}^{(j)}_*) = \max_{(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \sim \mathbb{P}} \alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big)$. For simplicity, let $A = u(\mathbf{x}^{(i)}) - u(\mathbf{x}^{(j)})$ and $B = K_{u|\mathcal{D}}(\mathbf{x}^{(i)}, \mathbf{x}^{(i)}) + K_{u|\mathcal{D}}(\mathbf{x}^{(j)}, \mathbf{x}^{(j)})$. We consider the following policies:

1. Upper confident bound on predictive preference (UCB) maximizes the 95% confidence interval of preference prediction: $\alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big) = \big|A + 1.96\sqrt{B}\big|$.
2. Differential entropy of the latent utility (DE-U) maximizes the log variance of utility posterior: $\alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big) = \frac{1}{2}\log(2\pi B) + \frac{1}{2}$.
3. Differential entropy of the marginal effects (DE-ME) maximizes the log variance of marginal effects approximated using our proposed GMM in Eq. (9):

$$\alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big) = \log\Big|\sum_{k\in\{i,j\}}\sum_{r=1}^{N} \omega_r \phi\big(\bar{f}_r(\mathbf{x}^{(k)})\big)\phi\big(\bar{f}_r(\mathbf{x}^{(k)})\big)^T \circ \nabla^2 K_{u|\mathcal{D}}(\mathbf{x}^{(k)}, \mathbf{x}^{(k)})\Big|.$$

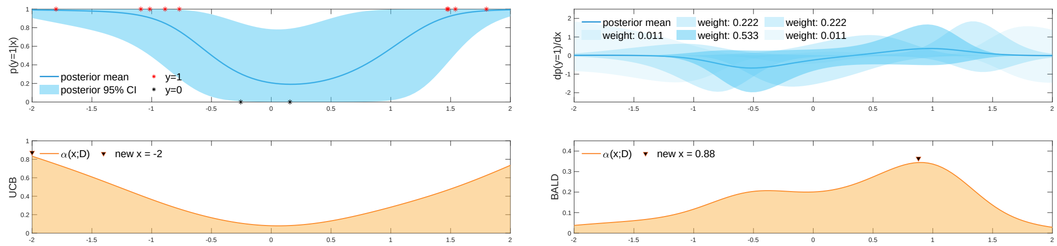


Figure 2: A 1-d example for illustrating acquisition functions of UCB and BALD. Upper panel shows the observed data with model posterior (left) and current marginal effect estimation (right). Lower panel shows acquisition value of UCB and BALD for selecting new profile, where the marginal effect variance at UCB's selection is low and that at BALD's selection is high. While UCB tends to exploit and optimize profile preference, BALD tends to explore and minimize model uncertainty.

4. Bayesian active learning by disagreement (BALD) aims to maximize the mutual information between the utility model and predictive preferences:

$$\alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big) = \mathbf{I}(y_{ij}, u; \mathbf{x}^{(i)}, \mathbf{x}^{(j)}, \mathcal{D}).$$

With entropy function $h(p) = -p\log(p) - (1-p)\log(1-p)$ and constant $C = \sqrt{\pi \log(2)/2}$, the approximated mutual information is:

$$\alpha\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)}); \mathcal{D}\big) \approx h\Big(\Phi\big(\frac{A}{\sqrt{B+1}}\big)\Big) - \frac{C}{\sqrt{B+C^2}} \exp\big(-\frac{A^2}{2(B+C^2)}\big).$$

5. Random sampling (UNIFORM) simply selects pairs uniformly at random from $\mathcal{P}$.

While UCB emphasizes *exploiting* current belief to find the most preferred profile, DE-U, DE-ME and BALD focus on *exploring* the profile space by reducing model uncertainty on either latent utility, marginal effects or the predictive preferences. Figure 2 shows a 1-d example for illustrating acquisition functions of UCB and BALD. Upper panel shows the observed data with model posterior (left) and current marginal effect estimation (right). Lower panel shows acquisition value of UCB and BALD for selecting new profile, where the marginal effect variance at UCB's selection is low and that at BALD's selection is high. This demonstrates that while UCB tends to exploit and optimize profile preference, BALD tends to explore and minimize model uncertainty.

# 6 Experiments

We first evaluate the estimated marginal effects by GPCA using synthetic data when the functional relations are known and could be computed analytically, and then consider adaptive experimentation of GPCA with several active learning policies. We also apply GPCA to two real-world data.

**Data generating process**. Following the simulation specification in Chu and Ghahramani [12], we consider two generating processes with discrete (2DPLANE) and continuous (FRIEDMAN) attributes.[1] The 2DPLANE dataset has 5 discrete attributes where $x_1 \in \{-1, 1\}$ and $x_2, \ldots, x_5 \in \{-1, 0, 1\}$, with a piecewise linear utility $u(\mathbf{x}) = 1 + 2x_2 - x_3$ if $x_1 = -1$ and $u(\mathbf{x}) = 1 + x_4 - 2x_5$ if $x_1 = 1$. The FRIEDMAN dataset has 3 continuous attributes where $x_1, \ldots, x_3 \sim [0, 1]$ with a non-linear utility $u(\mathbf{x}) = 3\sin(\pi x_1 x_2) + 6(x_3 - 0.5)^2$. We randomly sample 1000 pairs of profiles in each dataset and set $y_{ij} = 1$ with probability of $\Phi\big(u(\mathbf{x}^{(i)} - u(\mathbf{x}^{(j)})\big)$ and $y_{ij} = 0$ otherwise.

## 6.1 Accuracy of marginal effect estimation

**Evaluation metrics and baselines.** We consider three metrics for evaluation of both marginal effects and component effects: (1) the RMSE of the estimated effects, (2) the correlation (COR) between the estimated effects and true effects, and (3) the log likelihood (LL) of the estimated effects. We also compare our proposed GMM approximation for marginal effects in GPCA to several baselines: (1)

[1] See `https://www.dcc.fc.up.pt/~ltorgo/Regression/DataSets.html` for details.

Table 1: Averaged performance and standard deviations of both marginal and component effects from our GP-GMM estimator and baselines on the 2DPLANE and FRIEDMAN datasets. Models that perform statistically significantly better than all the others in paired t-tests are indicated in bold, while methods performing comparably to best models are indicated in italics.

| DATASET | ESTIMATOR | Marginal effects | | | Component effects | | |
|---|---|---|---|---|---|---|---|
| | | RMSE ↓ | COR ↑ | LL ↑ | RMSE ↓ | COR ↑ | LL ↑ |
| 2DPLANE | DIM | 0.712±0.022 | 0.013±0.003 | −2.137±0.115 | 0.109±0.005 | 0.341±0.029 | 0.494±0.117 |
| | LM-GMM | 0.213±0.001 | 0.340±0.005 | −0.238±0.145 | 0.069±0.002 | 0.475±0.019 | −0.778±0.157 |
| | GP-MAP | 0.175±0.002 | 0.732±0.007 | −3.893±0.863 | 0.052±0.002 | *0.611±0.024* | 1.401±0.177 |
| | GP-GMM | **0.135±0.002** | **0.803±0.007** | **0.563±0.023** | **0.044±0.001** | **0.616±0.025** | **2.000±0.082** |
| FRIEDMAN | DIM | 0.910±0.008 | 0.024±0.005 | −9.658±0.392 | 0.150±0.010 | 0.944±0.017 | −1.824±0.480 |
| | LM-GMM | 0.845±0.010 | 0.328±0.007 | −1.001±0.271 | 0.078±0.005 | 0.980±0.005 | 0.503±0.245 |
| | GP-MAP | 0.510±0.008 | *0.830±0.006* | −3.869±0.530 | **0.042±0.003** | **0.995±0.001** | *1.680±0.045* |
| | GP-GMM | **0.478±0.008** | **0.847±0.005** | **−0.213±0.065** | **0.042±0.003** | **0.995±0.001** | **1.689±0.044** |

the non-parametric diff-in-mean estimator (DIM) [1], where the continuous attributes in FRIEDMAN are first discretized by splitting into equally-spanned intervals, (2) the standard preference learning method with linear utility (LM-GMM) [9, 10, 8, 11], and (3) an ablated GPCA method (GP-MAP) but with MAP estimation of marginal effects.

**Results.** We repeat our simulation with 25 different random seeds using 300 Intel Xeon 2680 CPUs. Table 1 shows the averaged performance and standard deviations (STDs) of both marginal effects $\pi\big((\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\big)$ and component effects $\pi_l(\mathbf{x}_l^{(i)})$ defined in Eq. (4 and 7) from our GP-GMM estimator and baselines on the 2DPLANE and FRIEDMAN datasets. Models that perform statistically significantly better than all the other models in paired t-tests are indicated in bold, while methods performing comparably to the best method are indicated in italics. Our proposed GP-GMM leads to more precise effect estimation with lower RMSE and higher COR/LL for both marginal and component effects. In addition, Table 2 shows the averaged accuracy and STDs of preference prediction from GPCA and baselines on both synthetic datasets. GPCA has the best prediction for capturing the underlying preferential relations in the system.

Table 2: Averaged accuracy and STDs of preference prediction from GPCA and baselines on both synthetic datasets. GPCA has the best prediction for capturing the underlying preferential relations in the system.

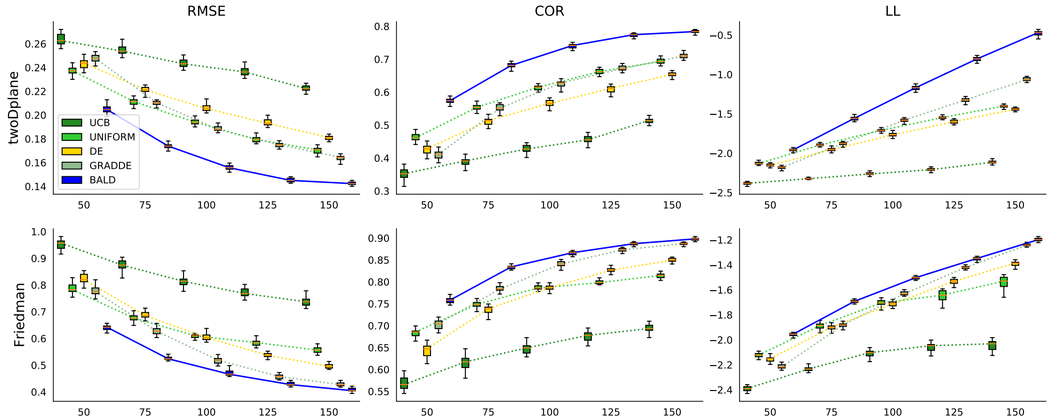
| DATASET | 2DPLANE | | | FRIEDMAN | | |
|---|---|---|---|---|---|---|
| | DIM | SVM | GPCA | DIM | SVM | GPCA |
| ACC | 0.696±0.006 | 0.824±0.003 | **0.986±0.002** | 0.785±0.006 | 0.795±0.005 | **0.956±0.002** |

### 6.2 Improved efficiency from adaptive experimentation

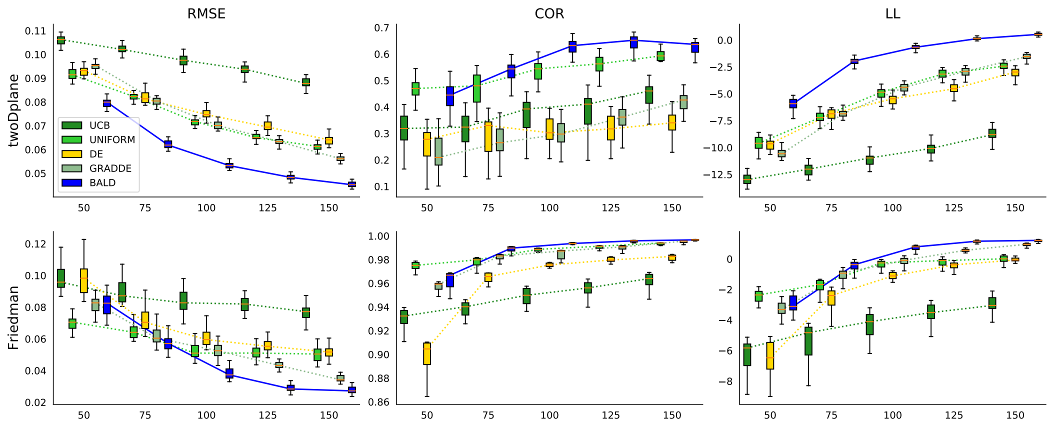
We then investigate adaptive experimentation in GPCA for increasing efficiency of effect estimation. We consider various policies: (1) UCB popular in multi-arm bandit setting [35], (2) DE-U and DE-ME for active learning based on differential entropy [36–38], (3) BALD in Bayesian active learning for model uncertainty reduction [39] and (4) UNIFORM design in non-parametric conjoint analysis [1, 2].

**Experimental details.** We initialize all the policies with the same 25 profile pairs from the 1000 candidate pairs, and update model posterior in GPCA once new preferences are revealed. Since the sampled profile distributions from each policy differ from each other due to their adaptive essence, we estimate the marginal and component effects w.r.t the same target profile distribution to ensure comparability. Specifically, we train our GPCA model on revealed preferences from profile pairs acquired so far and estimate both effects using GP-GMM at all the 1000 pairs.

**Results.** Figure 3 shows box plots of averaged RMSE, COR and LL and their STDs of marginal (top panel) and component (bottom panel) effects with adaptive experimentation under different acquisition policies. Sample size range from 50 to 150, and performance metrics are reported every other 25 acquisitions. Overall BALD (blue) outperforms the rest of policies including UNIFORM and UCB, indicating higher efficiency for effect estimation when the acquisition is designed to reduce




(a) Marginal effects




(b) Component effects

Figure 3: Box plots of averaged RMSE, COR and LL and their STDs of marginal (top panel) and component (bottom panel) effects with adaptive experimentation under different acquisition policies. Sample size range from 50 to 150, and performance metrics are reported every other 25 acquisitions. Overall BALD (blue) outperforms the rest of policies including UNIFORM and UCB, indicating higher efficiency for effect estimation when the acquisition is designed to reduce model uncertainty.

model uncertainty. Morever, UCB (forest green) has overall the worst performance in estimating both marginal and component effects as it solely reinforces current belief on the probability of preference.

**Preference prediction**. Besides estimation of marginal effects, we also examine the model quality of GPCA by evaluating the prediction accuracy of unrevealed preferences among the not acquired profile pairs. Figure 4 shows the averaged accuracy and STDs of preference prediction by various policies. With as few as $50$ data points, GPCA manages to predict at least $80\%$ of the unrevealed preference and $95\%$ when $150$ data points are adaptively acquired by BALD.

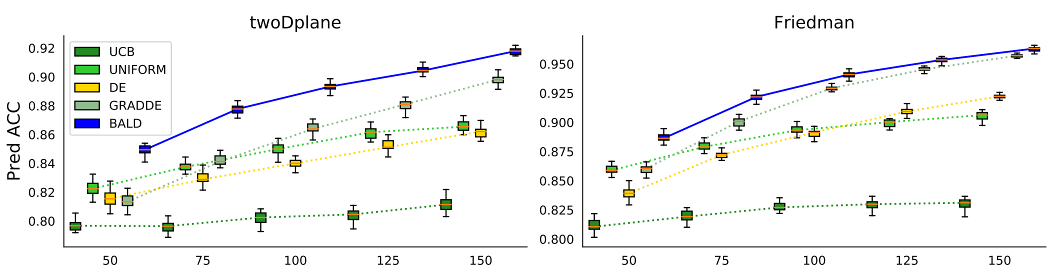



Figure 4: Averaged accuracy and STDs of preference prediction by various policies for simulated data.

### 6.3 Applications

**Data.** We apply GPCA to two real-world conjoint experiments: U.S. citizens' preferences across presidential candidates and attitudes toward immigrants containing 1733 and 6980 pairwise comparisons [1, 40]. Attributes in the candidate experiment include various aspects of candidates' personal background, demographics and issue positions, such as religion, education, profession, income and race, while attributes in the immigrant experiment include employment plans, job experience, language skills, country of origin, reasons for applying and so on.

Table 3: List of attributes with estimated component effects by GPCA and DIM used in the original studies, grouped by negative, null and positive effects.

| DATASET | GPCA \ DIM | NEG | NULL | POS |
|---|---|---|---|---|
| Candidate | NEG | Evangelical protestant, Mormon, car dealer,Age 68 | Jewish,Catholic,high school teacher, farmer,Income 210K,Black,Age 60 | — |
| | NULL | — | Mainline protestant,Lawyer,doctor,female, Income 54K,Hispanic,Asian American,Age 52 | Baptist college,Income 65K |
| | POS | — | Income 92K,5.1M,Caucasian, Native American,Age 45,75 | Military,community college, state university,Ivy League |
| Immigrant | NEG | India,China,will look for work, interview with employer,once as tourist | Broken English,Used interpreter,Germany, France,Mexico,Philippines,Poland,Iraq | — |
| | NULL | — | Mainline protestant,Lawyer,doctor,female, Income 54K,Hispanic,Asian American,Age 52 | — |
| | POS | — | Male,Somalia,financial analyst, waiter,child care provider | college degree,graduate degree,teacher,nurse,doctor computer programmer,research scientist,escape persecution |

**Results.** We run GPCA using all samples in both datasets. Table 3 shows the list of attributes with estimated component effects by GPCA and DIM used in original studies grouped by negative, null and positive effects. Overall, component effect estimation by GPCA is more reasonable. For example, in the candidate experiment GPCA found negative effects of Black candidates working as high school teachers or farmers on the probability of becoming U.S. presidents and positive effects of Caucasian candidates with 5.1M or more annual income, while DIM found no effects for any of these attributes. In the immigrant experiment, GPCA found negative effects of Iraqi applicants with broken English on the probability of immigration approval and positive effects of applicants working as financial analysts, while DIM found no effects. Figure 5 shows the averaged accuracy and STDs of preference prediction by various policies for real data with sample size varying from 100 to 800, where BALD has better prediction of unrevealed preferences than randomized policy.

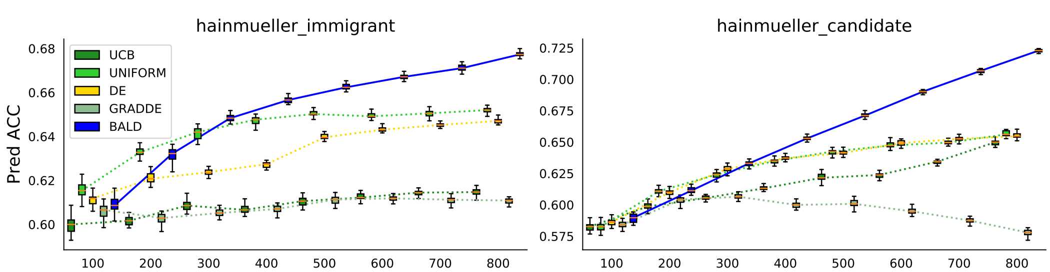



Figure 5: Averaged accuracy and STDs of preference prediction by various policies for real data, with sample size varying from 100 to 800. BALD has better prediction of unrevealed preferences than randomized policy.

## 7 Conclusion

We introduce GPCA, a Gaussian Process conjoint analysis model for estimating marginal effects in choice-based conjoint experiments. GPCA derives marginal effects as first-order derivatives and approximates their distributions using Gaussian mixtures, enhancing precision and efficiency in effect estimation aided by adaptive experimentation. GPCA has the potential of advancing causal inference in adaptive conjoint experiments. As distributional shifts are inevitable between adaptive acquired samples and uniformly randomized samples, directly interpreting marginal effects from adaptive samples in GPCA as causal effects may not be appropriate. Future research may explore methods such as inverse propensity weighting or doubly robust strategy for causal inference or feature interpretability in GPCA with adaptive experimentation.

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